

Renormalization of strongly coupled U(1) lattice gauge theories

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Abstract: Recent numerical studies of the 4D pure compact U(1) lattice gauge theory, I have participated in, are reviewed. We look for a possibility to construct an interesting nonperturbatively renormalizable continuum theory at the phase transition between the confinement and Coulomb phases. First I describe the numerical evidence, obtained from calculation of bulk observables on spherical lattices, that the theory has a non-Gaussian fixed point. Further the gauge-ball spectrum in the confinement phase is presented and its universality confirmed. The unexpected result is that, in addition to massive states, the theory contains a very light, possibly massless scalar gauge ball. I also summarize results of studies of the compact U(1) lattice theory with fermion and scalar matter fields and point out that at strong coupling it represents a model of dynamical fermion mass generation.

Talk presented at the *31st International Symposium Ahrenshoop on the Theory of Elementary Particles*, Buckow, September 2-6, 1997.

1 Introduction

Quantum field theories used in the standard model and its supersymmetric extensions are either asymptotically free or so-called trivial theories. Both are defined in the vicinity of Gaussian fixed points and can be studied perturbatively. In various collaborations we address the question whether in four dimensions (4D) there exist quantum field theories which are not accessible to or anticipated by perturbation theory. I describe the results of our systematic numerical study of critical behavior in several compact U(1) gauge models on the lattice, both pure and with matter fields. Their common feature is confinement when the bare gauge coupling is strong. It has never been clear whether this is only a lattice artefact or whether a confining continuum U(1) gauge theory with confinement can be constructed. Though not yet conclusive, our results are promising and we hope to stimulate more theoretical attention.

2 Renormalization of lattice field theories

In numerical simulations of lattice field theories the concept of renormalizability requires the existence of critical behaviour somewhere in the bare coupling parameter space. Dimensionful observables, e.g. masses m_s, m_1, m_2, \dots , n -point functions, etc, can be calculated in the lattice constant units a as functions of couplings in the vicinity of a critical point or manifold. The dimensionless correlation lengths $\xi_i = 1/am_i \rightarrow \infty$ when a critical point is approached.

Renormalizability further requires the existence of “lines of constant physics” in the bare coupling parameter space, along which the observables scale, i.e. their dimensionless ratios $r_1 = m_1/m_s, r_2 = m_2/m_s, \dots$, stay (approximately, in practice) constant. If such a line hits the critical point, one can construct the continuum limit and obtain the values of dimensionful observables by fixing one mass scale, m_s , in physical units. Then $a = 1/\xi_s m_s \rightarrow 0$ and $m_i = r_i m_s$. These results are usually universal, i.e. independent of the detailed choice of the coupling space and lattice structure, being governed by a limited number of fixed points.

The lines of constant physics can approach a critical manifold but, before hitting it, leave the parameter space. Or they can enter phase transitions of weak first order. In both cases the correlation lengths grow, but stay finite. Some inherent cut-off $\Lambda = \xi_s^{max} m_s$ remains. If it is large with respect to the physical scale m_s , i.e. if $\xi_s^{max} \gg 1$, the theory is renormalizable in a restricted sense. In this way e.g. the familiar trivial theories arise.

3 Pure gauge theory on spherical lattices

Recently we have reconsidered the oldest candidate for a non-Gaussian fixed point in the 4D lattice field theory, the phase transition between the confinement and the Coulomb phases in the pure compact U(1) gauge theory. We have used the extended

Wilson action in order to enlarge the possibility of finding a second order phase transition,

$$S = - \sum_P w_P [\beta \cos(\Theta_P) + \gamma \cos(2\Theta_P)]. \quad (1)$$

Here $w_P = 1$ and $\Theta_P \in [0, 2\pi)$ is the plaquette angle, i.e. the argument of the product of U(1) link variables along a plaquette P . Taking $\Theta_P = a^2 g F_{\mu\nu}$, where a is the lattice spacing, and $\beta + 4\gamma = 1/g^2$, one obtains for weak coupling g the usual continuum action $S = \frac{1}{4} \int d^4x F_{\mu\nu}^2$.

Earlier investigations performed as usual on toroidal lattices suggested that the phase transition, which is clearly of first order at positive γ , might be of second order at negative γ . Though a weak two-state signal is present there [1], it might be a finite size effect. It has been found that it disappears on lattices with sphere-like topology [2, 3, 4, 5]. As this type of lattice does not change the universality class [6, 7], one can use it as well as the usual toroidal one.

It has turned out that on spherical lattices the transition has properties typical for a second-order transition. This concerns, in particular, the dependence on the size of the lattice. Use of modern finite size scaling (FSS) analysis techniques, and large computer resources, allowed to determine on finite lattices the critical properties of the phase transition. A more detailed account of our work, as well as relevant references, can be found in Refs. [3, 4]. Here I list only the most important findings.

The measurements have been performed at $\gamma = 0, -0.2, -0.5$ on lattices of the volumina up to about 20^4 . The FSS behavior of the Fisher zero, specific heat, some cumulants and pseudocritical temperatures gave consistent results. The value of the correlation length critical exponent ν has been found in the range $\nu = 0.35 - 0.40$.

The most reliable measurement of ν has been provided by the FSS analysis of the Fisher zero, i.e. of the first zero z_0 of the partition function in the complex plane of the coupling β . The expected behavior with increasing volume V is $\text{Im } z_0 \propto V^{-1/D\nu}$. The joint fit to the data at all three γ values gives $\nu = 0.365(8)$. The scaling behavior of various pseudocritical temperatures, which have been determined from several other observables, is consistent with this value, as seen in Fig. 1.

The consistency of the data with the FSS theory suggests that the phase transition is of second order, as it implies growing correlation lengths. Unfortunately, it does not exclude that this growth stops on lattices substantially larger than those we have used. Very weak first order phase transition is still possible [8, 9], which would mean that the cut-off cannot be made really infinite. The scaling behaviour could be governed by a fixed point “behind” the phase transition.

In a very recent paper (which appeared after the Symposium) [10] Campos et al. suggest that this is what happens. Of course, also their data require an extrapolation to the infinite volume. For example latent heat, associated with the two-state signal present on toroidal lattices, decreases with volume. It can be extrapolated both to nonvanishing values and zero, depending on the ansatz. At some second-order transitions, two-state signals due to finite size effects are known to vanish extremely slowly with volume, mocking up a first order. It is most probably not possible to clarify the issue beyond any doubt by numerical methods. But the theory might be

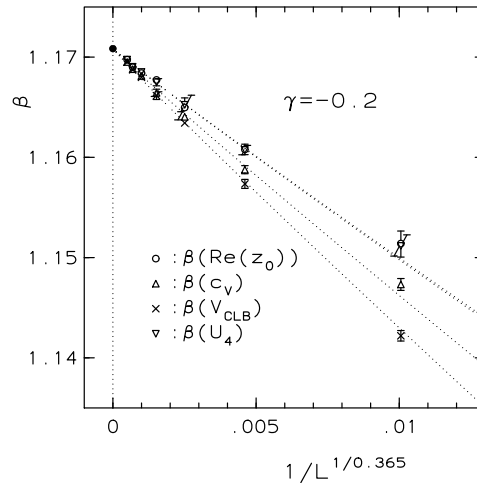


Figure 1: Consistency of the FSS of various pseudocritical temperatures with the value $\nu = 0.365$ of the correlation length exponent ν at $\gamma = -0.2$. The figure is from Ref. [4]

interesting even if the cut-off could not be made infinite but only much larger than the physical scale, as in physical applications we are dealing with effective theories anyhow.

4 Gauge-ball spectrum in the confinement phase

In any case the scaling behaviour of as many observables as possible should be determined. Therefore we have investigated at $\gamma = -0.2$ and -0.5 the spectrum of the theory in the confinement phase [11, 12]. It consists of massive “gauge balls”, states analogous to glue balls in pure QCD. In particular, no massless photon is present in this phase. Also the string tension σ has been estimated.

The gauge-ball masses m_j in various channels j of the cubic group have been measured at both γ for different β . Then their scaling behaviour in the form

$$m_j = c_j(\beta_c^j - \beta)^{\nu_j} \quad (2)$$

(and similar for $\sqrt{\sigma}$) has been determined in each channel j individually. We have found two groups of masses with strikingly different scaling behaviour. Within the whole β range a large group of the gauge-ball masses scale with roughly the same exponents ν_j close to the non-Gaussian value $0.365(8)$ found in [3, 4, 5]. Though its

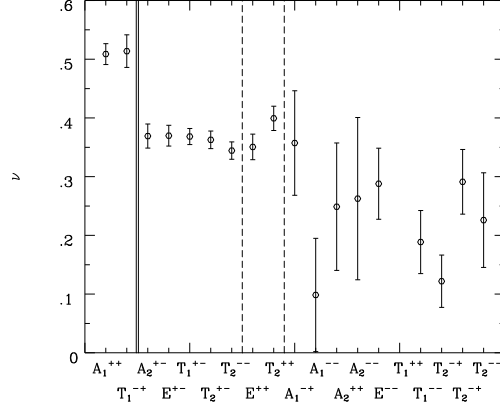


Figure 2: Values of ν_j obtained at $\gamma = -0.2$ in each gauge-ball channel separately. The double vertical line separates two groups with distinctly different ν_j . The results to the right of the dashed vertical lines correspond to heavier states which are very difficult to measure. The figure is from Ref. [13]

accuracy is not yet satisfactory, the exponent of $\sqrt{\sigma}$ seems to be consistent with this value, too. However, in several channels getting contribution from the 0^{++} gauge ball the values of ν_j are approximately Gaussian, i.e. $1/2$. This is shown in Fig. 2. The values β_c^j are quite consistent with each other in all channels for each γ . Therefore, in the further analysis we have assumed the same value of β_c in all channels.

Assuming the same exponent ν_j for each group, denoted ν_{ng} and ν_g for the non-Gaussian and Gaussian group, respectively, a joint fit was performed with these two exponents, common β_c , and the individual amplitudes c_j as free parameters:

$$m_j = c_j \tau^{\nu_f}, \quad f = \text{ng, g} \quad (3)$$

The resulting values of the exponents at $\gamma = -0.2$ are

$$\begin{aligned} \nu_{\text{ng}} &= 0.367(14) \\ \nu_g &= 0.51(3). \end{aligned} \quad (4)$$

The preliminary results at $\gamma = -0.5$ are consistent with these values [12].

From these results we conclude that the system has two mass scales which we denote by m_{ng} and m_g . In Fig. 3 we show their behaviour. Except 0^{++} , the gauge-ball masses and presumably $\sqrt{\sigma}$ scale proportional to m_{ng} . The ratios $r_j = m_j/m_{\text{ng}}$ are thus constant. This holds for both γ values. Thus, within the limits of numerical determination, we have found two lines of constant physics. We have further found that r_j are independent of γ , which indicates universality. This suggests that the scaling behaviour of the pure U(1) gauge theory belongs at different γ to the same universality class governed by one non-Gaussian fixed point. The parameter γ is irrelevant in this class as long as it is kept negative.

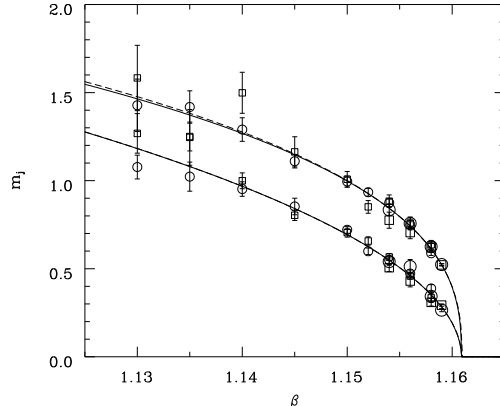


Figure 3: Masses of the 0^{++} (lower points) and 1^{+-} glue balls in the confinement phase versus β at $\gamma = -0.2$. The curves are fits (3) with the ν values (4). The figure is from Ref. [13].

Choosing m_{ng} for a physical scale, we can consider a continuum theory (or a theory with large cut-off) with the spectrum given by the values of r_j . They are given in [11]. As m_g/m_{ng} approaches zero, this theory would contain massless (or very light) scalar, which could possibly decouple.

What kind of theory would it be? The procedure we have applied to the spectrum and σ can be extended to all observables and, in general, to n -point functions. If scaling in corresponding powers of m_{ng} is found, one can, in principle, construct the theory without having any continuum Lagrangian for it. Or it could be some non-polynomial Lagrangian in the fields corresponding to the states in the spectrum. As σ would possibly be finite and nonvanishing in the m_{ng}^2 units, it would be a confining theory. An interesting proposal for an approximate continuum Lagrangian has been made in Ref. [14] and reported at this workshop by N. Sasakura.

The pure U(1) lattice gauge theory with the Villain (periodic Gaussian) action belongs to the same universality class [5]. Rigorous dual relationships valid for that action imply that also the following 4D models are governed by the same non-Gaussian fixed point: the Coulomb gas of monopole loops [15], the noncompact U(1) Higgs model at large negative squared bare mass (frozen 4D superconductor) [16, 17], and an effective string theory equivalent to this Higgs model [18, 19].

5 Compact U(1) gauge theory with matter fields

These findings raise once again the question, whether in strongly interacting 4D gauge field theories further non-Gaussian fixed points exist, which might possibly be of interest for theories beyond the standard model. The pursuit of this question requires an introduction of matter fields. Therefore we have investigated some extensions of

the pure compact U(1) gauge theory which might have interesting fixed points. We introduce fermion and scalar matter fields of unit charge, either each separately or both simultaneously. Because of space limits I give only a very brief overview of the results.

The compact fermionic QED has some analogous promising properties in the quenched approximation [20]. In compact scalar QED we confirm the Gaussian behavior at the endpoint of the Higgs phase transition line [21]. In a theory with both scalar and fermion matter fields ($\chi U\phi$ model), the chiral symmetry is broken, and the mass of unconfined composite fermions $F = \phi^\dagger \chi$ is generated dynamically [22]. It might be an explanation or alternative to the Higgs-Yukawa sector of the SM. In 2D [23] and 3D [24] it belongs very probably to the universality class of the 2D and 3D Gross-Neveu model, respectively, and is thus nonperturbatively renormalizable. In the 4D $\chi U\phi$ model we demonstrate the existence of a tricritical point where the scaling behaviour is distinctly different from the four-fermion theory, enhancing the chances for renormalizability [22, 25, 21]. Here, apart from the massive fermion and Goldstone boson, the spectrum contains also a massive 0^{++} gauge ball. The particular role of this state both in pure U(1) and in the $\chi U\phi$ model is remarkable and needs a theoretical explanation.

I thank J. Cox, W. Franzki, C. B. Lang, T. Neuhaus, A. Seyfried, and P.W. Stephenson for collaboration on the topics I have described, and J. Ambjorn, I. Campos, and H. Pfeiffer for discussions. The computations have been performed on the computers of HLRZ Jülich, RWTH Aachen, and KFU Graz. Our work was supported by Deutsches BMBF and DFG.

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